Short Papers

A Simple Evaluation of Losses in Thin Microstrips

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***Abstract*—A simple modification of Wheeler's incremental inductance rule is presented which allows the extention of the use of this rule for the evaluation in quasi-TEM operation of losses in thin microstrips (i.e., when the Wheeler's rule is considered no longer applicable). A good agreement of the proposed formula with available numerical results is obtained when thickness is comparable with skin depth. A comparison is also made with two approximate formulas proposed by some authors. The proposed modification to Wheeler's rule should be useful for computer-aided design (CAD) of monolithic microwave integrated circuits (MMIC's).**

I. Introduction

In this paper it is shown that it is possible to introduce a simple modification in the Wheeler's rule to evaluate losses in thin microstrips (at least in the quasi-TEM approximation and in absence of anomalous skin effect) when the metallization thicknesses t range in the skin depth's δ order of magnitude as in monolithic microwave integrated circuits (MMIC's) (i.e., when $t/\delta < 5$). A good agreement is found with the calculations performed with sophisticated methods as shown in [1]–[9], which require long numerical computations and are not appropriate for CAD implementations. The procedure is presented in Sections II and III for microstrips, but it may be extended to other thin-strip structures (e.g., strip lines, coplanar strips, etc.). In Section IV the results obtained with this approximate method are compared with both the few available numerical results and with those obtained with two approximate empirical formulas given in [3] and [5], [6].

II. Description of the Wheeler's Rule Modification

Let w, t, t_g, and h be the microstrip dimensions [defined in Fig. 1(a)], ε_r the dielectric relative permittivity of substrate and $\varepsilon_{r,\text{eff}}$ its effective value, σ the metal conductivity, δ the skin depth, $R_s = 1/\sigma\delta, Z_s = R_s(1+j)$ the metal wave impedance and $\gamma_c = (1+j)/\delta$ the metal propagation constant, $\eta = \sqrt{\mu_0/\varepsilon_0}$ the vacuum wave impedance, Z_c the lossless microstrip characteristic impedance, $F_z = Z_c\sqrt{\varepsilon_{r,\text{eff}}}/\eta$ the form factor of Z_c, and $j\beta_{\text{eff}} = j\omega\sqrt{\mu_0\varepsilon_o}\sqrt{\varepsilon_{r,\text{eff}}}$ the microstrip lossless propagation constant. The microstrip attenuation α due to conductor losses may be computed as the real part of the microstrip propagation constant γ, which can be expressed as a function of $j\beta_{\text{eff}}$, Z_c, and Z the conductor impedance per unit length, due to the field penetration in metal

$$\gamma = \alpha + j\beta = j\beta_{\text{eff}}\sqrt{1 - j\frac{Z}{\beta_{\text{eff}} Z_c}}. \tag{1}$$

In case the Wheeler's rule can be applied (thick microstrips and quasi-TEM behavior), the impedance Z, as known [10], is given by

$$Z = Z_s\frac{\partial F_z}{\partial n} = Z_w + Z_t + Z_g \tag{2}$$

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where Z_w is the contribution to Z of the strip large sides, Z_t that of the strip lateral sides, and Z_g that of the ground plane

$$Z_w = \left(\frac{\partial F_z}{\partial h} - 2\frac{\partial F_z}{\partial t}\right)Z_s \quad Z_t = -2\frac{\partial F_z}{\partial w}Z_s$$
$$Z_g = \frac{\partial F_z}{\partial h}Z_s. \tag{3}$$

For a thin microstrip (always in quasi-TEM behavior), the proposed procedure replaces the value of Z in (1) with a new value Z_{eff} obtained by increasing Z_w, Z_t, and Z_g of suitable factors F_w, F_t, and F_g

$$Z_{\text{eff}} = Z_w F_w + Z_t F_t + Z_g F_g = Z_{\text{strip}} + Z_g F_g \tag{4}$$

where

$$Z_{\text{strip}} = Z_w F_w + Z_t F_t. \tag{5}$$

The factors F_w, F_t, and F_g to be introduced in (4) are given by (see Section III)

$$F_w = \coth(\gamma_c t) + \frac{2m}{1+m^2}\,\frac{1}{\sinh(\gamma_c t)}$$
$$F_g = \coth(\gamma_c t_g), \quad F_t = \coth\left(\gamma_c\frac{w}{2}\right) \tag{6}$$

where m is

$$m^2 = \frac{-\dfrac{\partial F_z}{\partial t}}{\dfrac{\partial F_z}{\partial h} - \dfrac{\partial F_z}{\partial t}}. \tag{7}$$

In case of thick ground metallization $(t_g \gg \delta)$ and large strips $(w \gg \delta)$, F_g, and F_t simplify in $F_g = 1$ and $F_t = 1$.

The behavior of Z_{eff}, as given by (4), is in good agreement in the microwave frequency range (i.e., typically when $t/\delta > 1$), with the available numerical results, as shown in Section IV. Comparisons are also made with the two quoted approximate formulas, which were obtained by forcing in an empirical way the value of Z_{strip} to $R_{\text{dc}} = 1/(\sigma w t)$ when $t/\delta < 1$.

This first approximate formula [6, eq. (19)] refers only to the case of an isolated strip conductor and is valid only in the restricted range of values presented in [6] (i.e., $w/t < 6$ and $\sqrt{2\mu\sigma f w t} < 8$).

The second approximate formula given in [3, eq. (3)] and [5, eq. (7)] (called in [5] the phenomenological loss equivalence method) consists simply in replacing in (1) Z [as given by (2)], with a new value Z', chosen to force Z' from Z to R_{dc} when $Z < R_{\text{dc}}$. This last formula, however, does not take into account the loss contribution of the ground plane, also when it is not negligible $(w > h, t_g \cong t)$.

The behavior of Z_{eff} of (4) at very low frequencies, below the microwave range $(t/\delta \ll 1)$, presents a strip contribution Z_{strip} to Z_{eff} [in (5)], which is slightly smaller than R_{dc} when $w/t < 6$ and is slightly greater than R_{dc} when $w/t > 9$. Due to the small difference between Z_{strip} and R_{dc} it is possible to improve the behavior of Z_{eff} up to very low frequencies by multiplying Z_{strip} in (4) by a suitable factor T, empirically chosen (as done in the previously quoted formulas given in [3] and [5]–[6]) to force the value of Z_{strip}

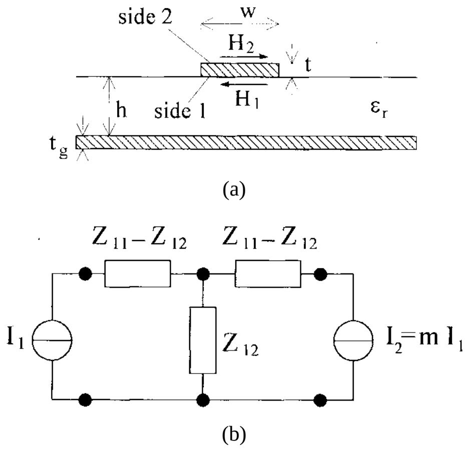

Fig. 1. (a) Microstrip configuration. (b) Equivalent circuit of the strip between sides 1 and 2 (a very large internal impedance has been assumed for the two current generators I_1 and I_2, being the metal wave impedance Z_s very small with respect to the vacuum wave impedance η).

to $R_{\rm dc}$ when t/δ tends to 0

$$Z_{\rm eff} = Z_{\rm strip} T + Z_g F_g. \tag{4'}$$

The factor T could be chosen modifying formula (7) of [5] in a suitable way to better match the available numerical results at low frequencies

$$T = [\coth(r)^3]^{1/3}\ \coth\left[y\left(\frac{t}{\delta}\right)^2\right]\ \tanh\left[\left(\frac{t}{\delta}\right)^2\right] \tag{8}$$

where (if $w/h < 10$)

$$r = {\rm Re}\{Z_{\rm strip}/R_{\rm dc}\}, \quad y = 1 + 0.013\ \ln^2\left(\frac{w}{t}\right)\left(1 - 0.1\frac{w}{h}\right).$$

III. Derivation of F_w, F_t, and F_g

The approximation involved in deriving the factor F_w (in addition to the quasi-TEM approximation) mainly consists in evaluating the magnetic energy and the power lost in the strip conductor, associated to the magnetic fields [Fig. 1(a)] at the strip lower-side (H_1) and at the strip upper-side (H_2) as if the field were constant on the two strip sides and equal to the average spatial rms values H_{e1} and H_{e2}

$$H_{e1}^2 = \frac{1}{w}\int_0^w |H_1|^2\ ds, \quad H_{e2}^2 = \frac{1}{w}\int_0^w |H_2|^2 ds.$$

The parameter m^2 is defined as $m^2 = |H_{e2}|^2/|H_{e1}|^2$. The value m^2 can be evaluated as the ratio between the conductor losses in the strip sides 2 and 1, respectively, which can be obtained, as in (7), by means of the Wheeler's rule.

In case of constant magnetic fields ($H_1 = H_{e1}$ and $H_2 = H_{e2}$) on the strip sides 1 and 2, the input wave impedances $Z_{\rm in(1)}$ and $Z_{\rm in(2)}$ on the metal surfaces 1 and 2, respectively, can be obtained from the two port equivalent circuit of Fig. 1(b) of a transmission line model of the strip between sides 1 and 2. Therefore, they have the following values (instead of Z_s, which is adequate for $t \gg \delta$)

$$Z_{\rm in(1)} = Z_{11} + mZ_{12} = Z_s F_1$$
$$Z_{\rm in(2)} = Z_{11} + Z_{12}/m = Z_s F_2$$

being Z_{11} and Z_{12} the impedance matrix elements of the equivalent circuit of Fig. 1(b)

$$Z_{11} = Z_s\ \coth(\gamma_c t) \quad Z_{12} = Z_s/\sinh(\gamma_c t). \tag{9}$$

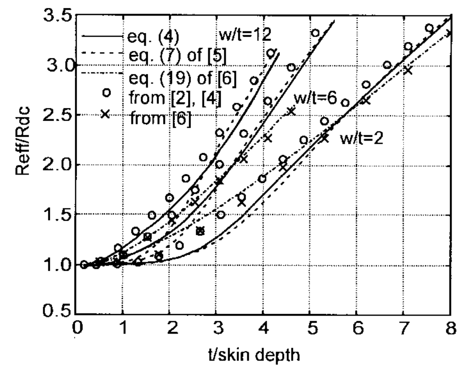

Fig. 2. AC resistance $R_{\rm eff} = {\rm Re}\{Z_{\rm eff}\}$ of a metallic strip of rectangular cross section, normalized to the dc strip resistance $R_{\rm dc}$ versus t/δ as obtained from [2], [4], and [6]. The numerical results are compared with those obtained from (4') by assuming h/w very large and with those obtained with approximate formulas [6, eq. (19)] and [5, eq. (7)].

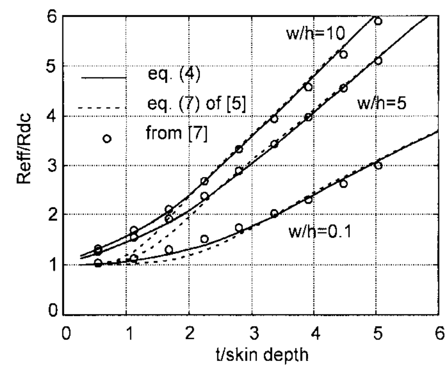

Fig. 3. Real part $R_{\rm eff}$ of $Z_{\rm eff}$ in a microstrip (normalized to the dc strip resistance $R_{\rm dc}$) versus t/δ, for $w/t = 5, t_g \gg t$ and for $w/h = 10, 5, 0.1$. The value given by (4') is compared with the numerical results computed in [7] and with the values obtained with the approximate formula [5, eq. (7)].

The sum of the flux of Poynting's vector on the strip side 1 and of that on the strip side 2 can be written as

$$\frac{1}{2}(Z_{\rm in(1)}|H_{e1}|^2 + Z_{\rm in(2)}|H_{e2}|^2)w = \frac{1}{2}Z_s|H_{e1}|^2 w(1+m^2)\frac{F_1 + m^2 F_2}{1+m^2} = PF_w. \tag{10}$$

In (10) $P = \frac{1}{2}Z_s|H_{e1}|^2 w(1+m^2)$ represents the flux per unit length of the Poynting's vector, evaluated in case of $\delta \ll t$ by means of Wheeler's rule; the factor F_w

$$F_w = \frac{F_1 + m^2 F_2}{1 + m^2} \tag{11}$$

takes into account the effect of the thin thickness and can be written as in (6). The other two factors F_g and F_t are derived in a similar way, being $m = 0$ for F_g and $m = 1$ for F_t.

IV. Comparison with Numerical Results

Figs. 2–5 compare the results obtained with the procedure here described with those obtained with the approximate empirical formulas [6, eq. (19)] and [5, eq. (7)] and available numerical results. The results with (4) and (4') have been obtained by using [11, eqs. (3.44) and (3.52)] for Z_c and [12, eq. (3.56)] for $\varepsilon_{r,\rm eff}$.

Figs. 2 and 3 present the behavior of $Z_{\rm eff}$, by varying the frequency (i.e., δ), for given values w/h and w/t; Figs. 4 and 5 present the behavior of a fixed frequency by varying t for fixed values of w and h (i.e., $w/h = 2$ in Fig. 4 and $w/h = 0.15$ in Fig. 5).

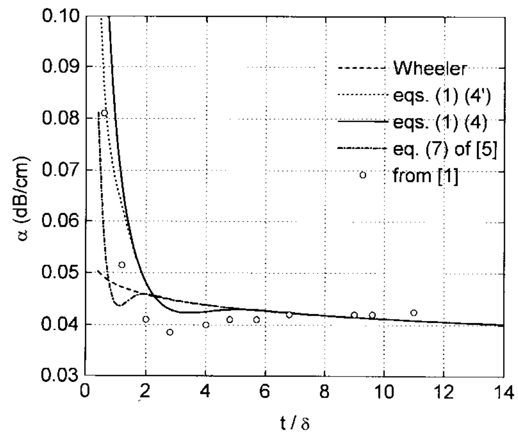

Fig. 4. Microstrip conductor loss in dB/cm at 8 GHz versus t/δ (metal thickness/skin depth) for $w/h = 2, h = 508\ \mu\text{m}, \varepsilon_r = 11, \sigma = 5.7 \cdot 10^7$ S/m. The results obtained from (1) with Z_{eff} from (4) and (4') are compared with those presented in [1] and with those obtained with the approximate formula [5, eq. (7)] and with Wheeler's rule.

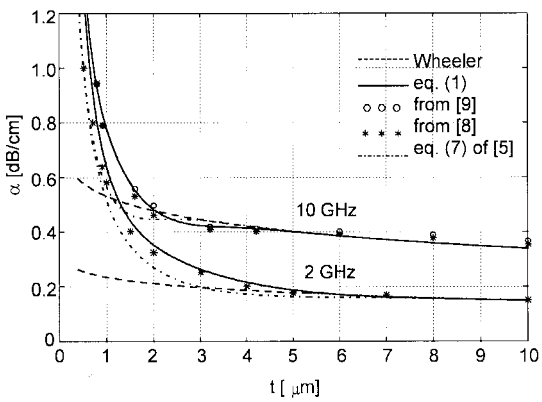

Fig. 5. Microstrip conductor loss in dB/cm at 2 GHz and 10 GHz versus t (metal thickness in micron) for $w = 30\ \mu\text{m}$, $h = 200\ \mu\text{m}, \varepsilon_r = 12.9, \sigma = 3.333 \cdot 10^7$ S/m. The results obtained from (1) with Z_{eff} from (4) are compared with those presented in [8] and [9] and with those obtained with the approximate formula [5, eq. (7)] and with Wheeler's rule. The skin depth δ results $\delta = 0.872\ \mu\text{m}$ at 10 GHz and $\delta = 1.95\ \mu\text{m}$ at 2 GHz.

Fig. 2 shows the behavior of the real part R_{eff} of Z_{eff} of an isolated metallic rectangular strip. The value of R_{eff}, evaluated with (4'), normalized to R_{dc}, vs t/δ, is compared with the available numerical results, computed in [2] (as presented in [4, Th. I]) and in [6], as well as with the behavior of the two approximate formulas [6, eq. (19)] and [5, eq. (7)]. Formula (19) of [6] is presented only in its range of validity $(w/t \leq 6)$.

The case of an isolated metallic rectangular strip is, however, not very significant because the accuracy of formulas used for the evaluation of Z_c can be poor when $w/h \rightarrow 0$. As shown in Fig. 3, the agreement of (4') with numerical results is much better for a microstrip while the underestimate of the values given by the approximate formula [5, eq. (7)] with respect to numerical methods is still more remarkable. The divergence increases when the losses in the ground plane become more important, i.e., by increasing w/h.

Other results for a microstrip are shown in the graphs of Figs. 4 and 5 in a large range of w/t values. In Fig. 4 the attenuation values obtained from (1) with Z_{eff} given both by (4') and by (4) [i.e., without the correcting factor T of (4')] by varying the thickness t with fixed values of w and h are compared with the values obtained from the approximate formula [5, eq. (7)] and with those obtained by the Wheeler's rule, as well as with the numerical results evaluated in [1] at 8 GHz for $w/h = 2$ ($\varepsilon_r = 11$, $w = 1016\ \mu$m, $h = 508\ \mu$m, $s = 5.7 \cdot 10^7$ S/m). A similar comparison is presented in Fig. 5 with the numerical results of [8] and [9] at 2 GHz and 10 GHz for a small width/height ratio, i.e., $w/h = 0.15$ ($w = 30\ \mu$m, $h = 200\ \mu$m, $\varepsilon_r = 12.9$, $\sigma = 3.333 \cdot 10^7$ S/m). In this case, curves with (4') are practically coincident with (4).

It can be observed that the differences between the approximate approach presented here and the numerical methods [1], [2], [7]–[9] are of the same order of magnitude of the differences between these sophisticated methods. The results are in particular very close with the most recent results [7]–[9].

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